

A Finite Element Beam-model for Efficient Simulation of Large-scale Porous Structures

MARTIN STAUBER, MARTIN HUBER, G. HARRY VAN LENTHE, STEVEN K. BOYD* and RALPH MÜLLER†

Institute for Biomedical Engineering, Swiss Federal Institute of Technology (ETH) and University of Zürich, Moussonstrasse 18, 8044 Zürich, Switzerland

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This paper presents a new method for the generation of a beam finite element (FE) model from a three-dimensional (3D) data set acquired by micro-computed tomography (micro-CT). This method differs from classical modeling of trabecular bone because it models a specific sample only and differs from conventional solid hexahedron element-based FE approaches in its computational efficiency. The stress–strain curve, characterizing global mechanical properties of a porous structure, could be well predicted ($R^2 = 0.92$). Furthermore, validation of the method was achieved by comparing local displacements of element nodes with the displacements directly measured by time-lapsed imaging methods of failure, and these measures were in good agreement. The presented model is a first step in modeling specific samples for efficient strength analysis by FE modeling. We believe that with upcoming high-resolution *in-vivo* imaging methods, this approach could lead to a novel and accurate tool in the risk assessment for osteoporotic fractures.

Keywords: Finite element method; Micro-compression; Bone architecture; Trabeculae; Image guided failure assessment

INTRODUCTION

Osteoporosis, which occurs most frequently in post-menopausal women and the aged, is defined as a metabolic bone disease characterized by low bone mass and micro-architectural deterioration of bone tissue, leading to enhanced bone fragility and a consequent increase in fracture risk [1]. It has been estimated that at least 20% of women have suffered one or more fractures by age 65, and as many as 40% suffer such fractures after the age of 65 [2]. It is increasingly recognized that osteoporosis is an important public health problem with rapidly escalating social and medical cost.

Densitometry use in the last three decades has resulted in a focus on bone density as the most important predictor of osteoporotic bone fractures for osteoporosis [3–5]. However, although many older persons may lose bone, as expressed by a decrease in bone density, not all develop fractures. This is not unexpectedly so, as bone density is not the sole determinant of fracture risk. Bone mineral

density, geometry of bone, micro-architecture of bone and quality of the bone material are all components that determine bone strength as defined by the bone's ability to withstand loading. For this reason, micro-structural information must be included in the analysis to predict individual mechanical bone properties [6,7]. Preliminary data have shown that predicting trabecular bone strength can be greatly improved by including architectural parameters in the analysis [8–10]. However, the relative importance of bone density and architecture in the etiology of bone fractures, an issue also referred to as bone “quality”, is poorly understood.

Due to the large volume fraction of vertebral trabecular bone—over 90% of the load in lumbar vertebrae is carried by trabecular bone [11,12]—an extended understanding of the failure behaviour of vertebral trabecular bone is essential for estimating the risk of these spontaneous fractures. Osteoporotic fractures are typically defined as 20–25% reductions in vertebral height [13] which means that for an atraumatic fracture and

*Current address: Department of Mechanical and Manufacturing Engineering, University of Calgary, 2500 University Drive, N.W., Calgary, Canada T2N-1N4

†Corresponding author. Tel.: +41-1-632-4592. Fax: +41-1-632-1214. E-mail: mrueller@biomed.ee.ethz.ch

the assumption that bone yields at about 1% strain [14], bone tissue is damaged long before we actually start speaking of an osteoporotic fracture. Nevertheless, most of the studies in the literature as presented above deal with the pre-yield or pre-failure behavior of bone. Mechanical tests concentrated on reporting results before the yield point or ultimate strength is reached. Only few attempts have been made to assess and quantify the mechanical behaviour of bone beyond initial yielding and failure in compact [15,16] and in trabecular bone [17–19].

Pathologic anatomy may be assessed from bony fractures and relative displacements as observed in imaging studies. Recent advances in imaging methodologies permit three-dimensional (3D) analysis of bone architecture. At the forefront of these technologies is micro-computed tomography (μ CT) which is a technique for the non-destructive assessment of 3D bone architecture, and can be used for both static and pseudo-dynamic measurements. Recently, Müller *et al.* [20] developed a method for time-lapsed measurements of micro-structural bone under compression to assess load-induced buckling and bending as well as fracture initiation and propagation in the 3D bone matrix. This method—termed as image-guided failure assessment (IGFA)—allows simultaneous assessment of the 3D structure of porous materials, its global deformations, and its corresponding stress–strain values. This technique provides experimental measurements for understanding the plastic post-failure behaviour of porous structures and these data can serve as the basis for the development of new theoretical models.

Many mathematical models have been proposed to investigate the mechanical behaviour of trabecular bone through representation of the microstructure as well as possible with respect to its mechanical properties. Analytical models have been widely used to represent bone mechanical behaviour by exploiting generalizations in the trabecular structure. Purely analytical models have been developed, for example, using the theory of cellular materials [21–24] or a network of doubly tapered struts [25] to investigate trabecular microstructure. Others have incorporated FE analysis [26–34], and these models have been successful in identifying important failure properties of cancellous bone, for example, that bending and buckling are important failure modes. Analytic and semi-analytic models have incorporated more specific bone micro-structural properties [27–30], including the use of an open-celled matrix containing roughly spherical pores [28], anisotropic material properties with strong vertical cylindrical beam elements connected to weaker horizontal elements [27] and the inclusion of fluid representing marrow [30]. While these models have been successful in identifying important failure modes [26], and that failure occurs in bands [32], and even for models of bone remodeling [33,34], the limitation of these models is that they are generalized representations of trabecular bone.

In contrast to generalized mathematical models of trabecular microstructure, specimen specific models are also important and are essential for direct prediction of specimen bone failure properties. In this study, the focus is on development of models for specific samples for the prediction of bone strength on an individual basis. Although specimen specific models can be obtained using a solid FE [35,36] approach, such models have some practical disadvantages and limitations. These include primarily the large computational effort which can require computational times on super computers for several hours or even days and weeks [36,37]. Furthermore, such models are not well suited for parametric studies where the structure can be freely altered with respect to number of elements and various element properties. Overcoming these limitations could give a deeper insight in the failure mechanism of porous structures.

We propose here a beam finite element (beam FE) model to investigate and predict failure behavior of individual trabecular bone structures. The presented beam FE model is an abstraction of the porous object (trabecular bone or a suitable substitute for research development) composed of simple cylindrical beam elements representing the trabecular elements in the real structure. While not a perfect recreation of the bone micro-architecture, its simplified representation has the advantage of an enormous decrease in the number of elements hence substantially decreasing the computational effort for those problems. Furthermore, the elements within the porous structure can be addressed and modified separately permitting parametric studies in order to gain a better general understanding of the failure behavior of trabecular bone-like structures. We believe that with upcoming high-resolution *in-vivo* imaging methods this approach could lead to a novel and accurate tool in the risk assessment for osteoporotic fractures on a patient specific basis.

METHODS

Image-guided Failure Assessment

In order to visualize material failure on a micro-structural level, a novel micro-mechanical testing system was devised to measure unloaded and loaded porous materials directly in a micro-tomographic system (μ CT) [20,38]. The micro-mechanical testing system consisted of two major components, the micro-compression device (MCD) and the material testing and data acquisition (MTDAQ) system, designed to provide uniaxial compression, data acquisition, and strain locking of the specimen during imaging. The MCD was designed to house the test specimens and act as a transportable link between the mechanical testing and μ CT imaging steps. Its function was to hold the specimen, lock the applied strain to the specimen, record the applied load *via* an onboard

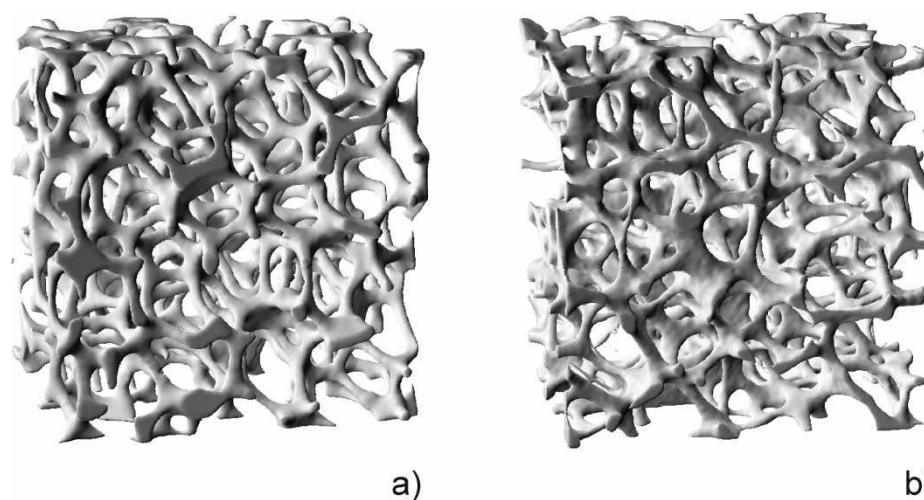


FIGURE 1 Two typical porous structures: (a) aluminum alloy, (b) vertebral trabecular bone. Due to the high similarity of those structures aluminum alloy is used as a surrogate for human trabecular bone.

load-cell, and provide a radiolucent window for scanning the specimen. The MCD has an outer diameter of 19 mm and a total length of 65 mm in order to fit in the μ CT opening. The internal load chamber fits wet or dry specimens with maximal diameter of 9 mm and maximal length of 22 mm. The MCD in combination with μ CT enables to simultaneously visualize bone failure and to assess its mechanical properties from the applied strain and the corresponding load. A detailed description of the device used can be found elsewhere [38].

For the purpose of this study, a single cubic aluminum alloy (6101-T6, ERG Oakland, CA) with a side length of 6 mm and a volume density of 8% was used. The porous alloy was used because its anisotropic and inhomogeneous nature is very similar to trabecular bone found in the human lumbar spine, which has an average volume density of $9 \pm 3\%$ [39]. The morphologic analogy between these two structures is best shown with three-dimensional visualizations (Fig. 1). It is noteworthy that these foams are comparable to bone in terms of their morphology, but not in terms of the mechanical properties of the aluminum alloys.

For data acquisition the samples were imaged and compressed at 0, 4, 8, 16 and 20% nominal strain (Fig. 2). The specimen was placed inside the MCD and preconditioned between 0.0 and 0.3% at a rate of 0.005 s^{-1} for 10 cycles to eliminate typical toe behavior [14]. Then, the specimen was transferred to the μ CT (μ CT 20, Scanco Medical AG, Bassersdorf, Switzerland)

for initial imaging of the sample in the intact state (0%). After imaging, the specimen was returned to the MTDAQ, and exposed to a first monocyclic nominal strain of 4% in order to capture the linear and yield behaviour of the material. This procedure was repeated three more times to acquire additional time-lapsed images of 8, 16 and 20% nominal strain in the post-failure regime. The acquisition time in μ CT for each image was approximately 90 min. From the displacement and load data acquired by the MTDAQ device a stress-strain curve was derived.

Generation of the FE Beam Model

The beam FE model was generated based on the reconstruction of the initial, uncompressed sample image. A three-dimensional Gaussian filtration with a sigma of 1.2 and a filter support of 1 voxel was applied to the original data to suppress noise in the image. This procedure was followed by a global segmentation technique based on the gray level histogram to separate object from background and to binarize the image. The threshold level was set to 28.6% of the image data range. The resulting binary volume served as the origin for all subsequent image processing procedures.

The next image processing step mapped each rod-like element within the structure to a beam element of the FE model using a spatial decomposition algorithm described earlier by Müller *et al.* [40]. This method capitalizes on the fact that a porous structure has increased thickness where



FIGURE 2 Image sequence of an aluminum foam as acquired by the image-guided failure assessment (IGFA) technique.

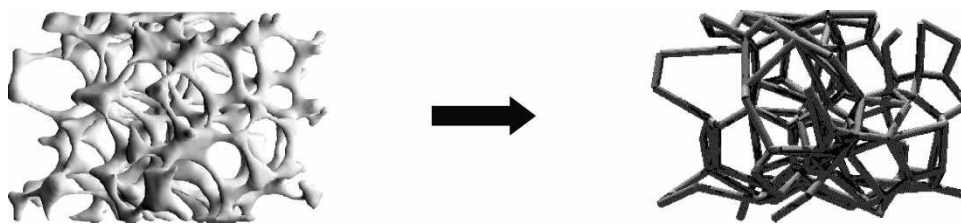


FIGURE 3 A spatial image decomposition algorithm enabled to map each element from the porous structure to a beam element within the FE model.

several elements join together. For the identification of those thicker spots first, a distance transformation (DT) was applied resulting in a new image where the values of the voxels corresponded to the shortest Euclidean distance to the phase interface [41]. By thresholding this volume only the thickest spots within the structure remained in the image. The mass centers of those spots were then defined to be the nodes in the corresponding beam FE model. The connectivity describing the elements was recovered from the DT volume using a so-called “turtle” algorithm [40]. At each node a “turtle” followed the mass centers of the DT image in all possible directions searching for neighboring nodes. Once another node was found, the points between the starting node and the end node were defined to be an element from the structure. A node and a connectivity list resulted from this algorithm describing the whole porous structure. This structure formed the underlying basis for a corresponding beam element FE model (Fig. 3).

The resulting FE model was imported into the commercial FE software package Marc/Mentat (MSC Software Corporation, Los Angeles, CA) where the shape and material properties of the elements were defined. To minimize model complexity we selected all elements to have the same geometry and the same material properties. The mechanical behavior was modeled with a two step linear function. Up to the yield point at 205 MPa the material was modeled to behave linear elastically with an elastic modulus of 65.7 GPa, beyond the yield point the material was modeled to behave ideally plastic with a hardening modulus of 165 MPa. These values were chosen based on the known characteristics of the aluminum alloy.

The beam elements were connected at the nodes. At each node, no relative rotation and translation between the two connecting beams was allowed. The upper and lower platens were fully fixed to the sample, which matches the experimental conditions. In each compressive step of the simulation an analyses was performed to check whether trabeculae get in contact with the upper or lower platen. A so called ‘glue option’ was active, indicating that once contact occurred there was no tangential motion between trabeculae and platen.

For our purpose, the best element type would be a beam with a full circular cross section. However, such an element does not exist in Marc [42]. Instead, we used an element (type 25) representing a straight, thin-walled, closed-section beam with circular cross-section. It is based on Euler–Bernoulli theory, allows for plastic deformation and

includes twist, which improves the element for large displacement analyses. When assigning appropriate (“equivalent”) dimensional and material properties, this element behaves as if it were a beam with full cross-section. To test our assumptions on the behaviour of this element, we performed a two step evaluation. First, a single beam composed of 10 two-noded beam elements was modeled. The beam was fixed at one end, and at the other end a load was applied; for this configuration a simple analytical solution is available. In the second step we integrated the beam elements in a spatial structure. The goal of this step was to evaluate the behaviour of the elements in a more realistic environment. The open cell cube model introduced by Gibson [21,43] seemed to be a good model for this purpose because of two reasons. First, this model was a good representation for our target structure, and second, for this model there existed an analytic solution which could be used to verify the FE simulation. The analytic solution for the relative Young’s modulus for the beam elements with quadratic cross section can be found to be [21]:

$$E^*/E_s \sim (t/l)^4$$

In this equation, E^* denotes the apparent Young’s modulus of the whole structure, E_s denotes the Young’s modulus of the structure material, t and l denotes the width and the length of the beam elements respectively, which compose the structure.

FE Analysis

For proof of principal, one bone analogue structure was modeled and solved as described above. The boundary conditions were chosen to simulate axial compression. The bottom platen was fixed, and the top platen applied compression by displacement control in steps of 0.2%. Any node coming in contact with either platen due to the compression were also constrained by the platen boundaries for all subsequent steps. The FE simulation of axial compression was done up to a strain of 3%.

RESULTS

Evaluation of the Beam Element Properties

To test our assumptions on the use of a thin-walled beam element in representing a full circular cross-section we

analyzed two configurations. In the first one, we modeled a single beam composed of 10 two-noded elements. This simple model showed the expected behaviour and the results were in excellent agreement with the analytic computation resulting in a deviation of the displacement at the end of the beam of less than 0.01%. In the second configuration, we used the model proposed by Gibson [21,43] to evaluate the beam properties in a more realistic environment. In this case the relative elastic modulus E^*/E_s as it was derived from the simulation was compared to the relative elastic modulus from the analytic computation for different relative densities ρ^*/ρ . Again, the results from our simulations were in excellent agreement with the analytic result with deviations between the two methods of less than 0.01%, indicating that this element is well-suited for our modeling purposes.

Aluminum Alloy

The stress–strain curve measured by IGFA has a steep slope in the first 0.8% of global strain, reaches its maximum at about 2% of global strain, and is followed by a flat plateau and a decrease in stress starting at about 5% of global strain (Fig. 4). The mechanical behaviour of the aluminum foam is therefore as expected and well described in literature [44]. The beam FE model proposed in this study was designed to predict the material behaviour up to the maximum value (ultimate load). This model could therefore only predict reasonable values up to approximately 3% of global strain. However, within this range the stress–strain curve computed by the beam FE model was in good agreement with the experimental data ($R^2 = 0.92$), and these results are shown in Fig. 4 providing the comparison of the experimentally measured (IGFA) and the theoretically computed (beam FE) stress–strain curves. This data illustrates that the presented model, despite its relative simplicity, is a good predictor of global failure behavior of porous structures.

Although the model was not designed to predict failure of single structural elements, the beam FE calculations of the displacements of most nodes were in good agreement

with the experimental data, at least qualitatively. A typical comparison of the displacement from the beam FE model and the measurement for four representative nodes within the structure illustrates this point (Fig. 5).

The use of a beam element model (718 elements) rather than solid hexahedron FE model (37,081 elements) allowed us to reduce the number of elements by a factor of 50. With that, also the average computation time could be reduced dramatically allowing non-linear analysis with a fast response time. The computation time for all iterations in the non-linear model was less than one minute on a HP Unix system (PA8600, 550 MHz, 4 GB of memory).

DISCUSSION

In this paper, we presented a new method for the generation of a beam FE model from a 3D data set acquired by micro-CT. This method differs from classical modeling of trabecular bone [21–34,43] in the fact that it models specific samples rather than generalizations, and it has the advantage over conventional solid FE approaches [35,36] to improve computational efficiency.

This model predicts global failure behaviour accurately through a relatively simplified FE model simulation. The simplicity of the model has two main advantages. First, the number of elements is reduced by a factor of 50 compared to a corresponding solid FE model. The reduction of the number of elements enables the use of commercial FE solvers. The scientific FE modeling community is large and covers multiple disciplines, therefore it can be advantageous for biomedical engineers to be able to benefit from these constantly improving commercial packages rather than developing customized FE codes on a laboratory-by-laboratory basis. Additionally, the computational time to solve the problem can be reduced dramatically. Second, the simplification of the model enables to easily modify certain parameters to investigate their influences on the mechanical behavior of the structure. For example, the influence of altered connectivity in the structure can be readily investigated by removing whole elements in the beam FE model.

Even though the model was not designed to predict local failure mechanism, the displacements of selected nodes were in good agreement with the measured node displacements. Uncertainties in the prediction of the new position of a node may arise from image processing and from the fact that all elements have the same geometry and material properties. Nevertheless, this model is promising for the local prediction of failure, and this is an ongoing area of research in our laboratory.

The method has been validated analytically by modeling two simple cases, a single beam composed of 10 elements and the open cellular model presented by Gibson [43], respectively. These two simple models were used since there was an analytic solution for both cases, which could

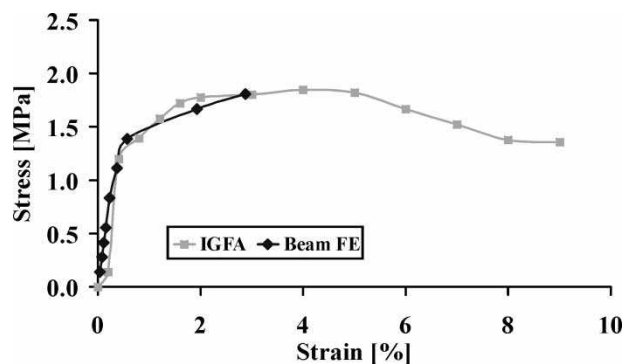


FIGURE 4 Stress–strain curves acquired by IGFA (gray) and the beam FE simulation (black). The two curves show that the beam FE simulation could predict the experimental data (IGFA) in the elastic and post yield region up to the maximal load.

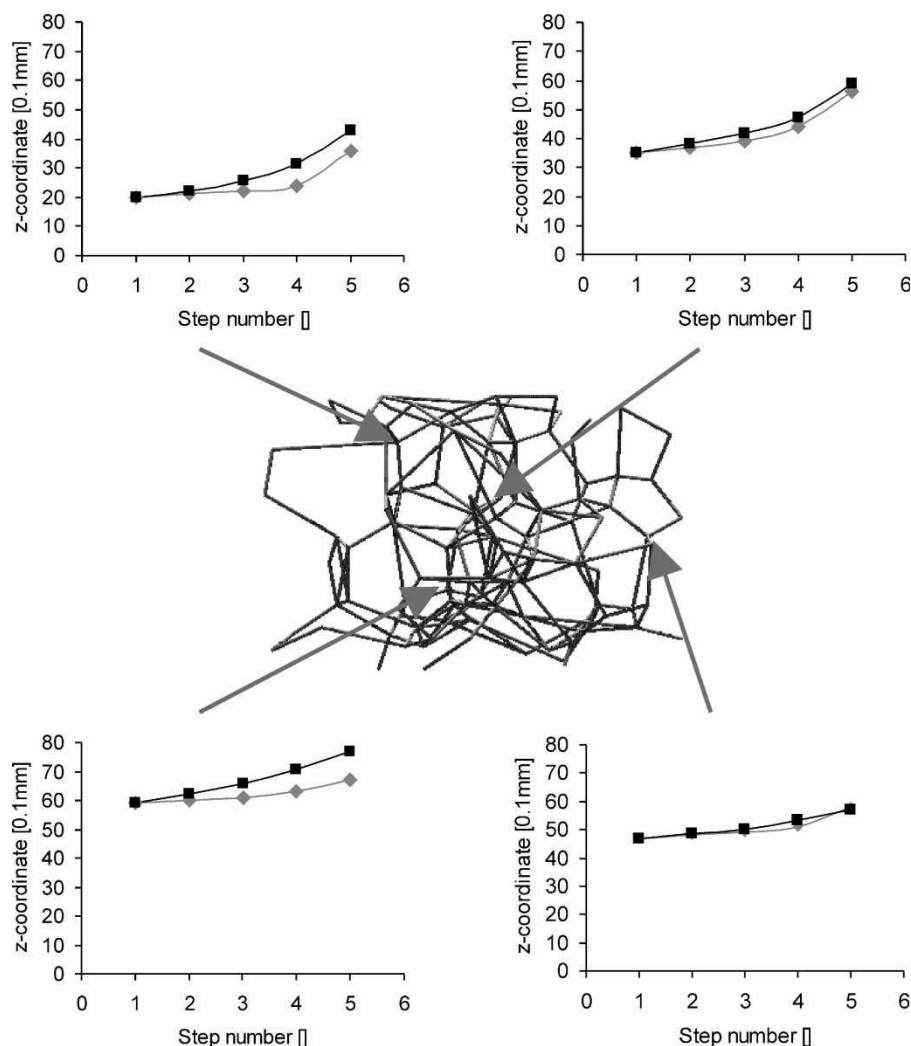


FIGURE 5 Comparison of the displacements of four selected nodes within the structure for the beam FE model (black) and the experimental data (gray). Although the model was not designed to predict local failure mechanisms, the predicted and experimentally measured displacements were well matched at most nodes.

be compared to the FE result. The displacements as calculated by FE were in excellent agreement with the analytic results.

Although the results from the bone analogue structure were also in good agreement with the actual measurement using IGFA, there were some limiting factors affecting the accuracy of the prediction of the global failure behaviour of the trabecular bone sample. One limitation was that a constant thickness of all beam elements was assumed. A second limitation was that plate-like structures have not been modeled. These limitations present challenges for the model to be successful for predicting material behaviours of increasingly complex and inhomogeneous structures. However, these limitations can be overcome by modifying the model and accepting a tradeoff between model simplicity and accuracy. Some beneficial modifications that minimize the increase in model complexity are described below.

A first modification for a better failure prediction could be done by mapping the mean thickness of each element

within the structure to a diameter of the corresponding beam in the FE model. The mean thickness can be assessed for each element by methods described elsewhere [39]. This individualization of the elements would not only allow better prediction of global failure behaviour, but would also possibly enable a more reliable prediction of the displacement of single nodes and hence prediction of failure behaviour of single elements. This method may allow to predict elements failing first and lead to a deeper understanding in the plastic post-failure mechanisms of porous structures.

Secondly, since the elements in the presented model were not designed to fail, the horizontal plateau and the negative slope in the stress–strain curve could not be predicted. To achieve this goal, the model could be modified in a way that elements where stress exceeds a certain value change their properties or even are completely removed from the model. With this modification, it might be possible to predict also the negative slope beyond the ultimate load of the stress–strain curve far beyond the yield point.

Third, in the presented model simple cylinders were used as beam elements. We propose two additional modifications to achieve a more realistic FE structure. First, a hyperbolic shape for the elements could be chosen as opposed to cylindrical. This could be approximated by mapping the inner parts of the six node elements to smaller radii. A modification of the elements in this sense would likely lead to an improved result since it includes the fact that nodes are thicker than the actual rods [25]. Second, a solution for plate-like structures should be found. This could be realized by introducing triangular shells representing plate-like elements. Complex plates could be subdivided into smaller triangular shells. Such a modification would allow using the model also for combined porous structures composed of plate and rod-like elements.

CONCLUSIONS

This study is a first step in developing a new model for failure prediction of porous structures. The presented model predicted a non-linear stress-strain curve that was in good agreement with the experimental data up to ultimate load. The main advantage of the model is its simplicity resulting in a short computation time and allowing easy parametric modifications that could yield in new models with even higher predictive power. Future models could be able to simulate complex porous structures such as trabecular bone, which are composed of rod- and plate-like elements. We believe that with upcoming high-resolution *in-vivo* imaging methods in patients this approach could lead to a novel and accurate tool in the risk assessment for osteoporotic fractures.

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